Abstract— As a non-weighted number system, the arithmetic operations in Residue Number System (RNS) are split into smaller parallel operations which are operates in free lance. There is no carry propagation between these operations. Hence devices operating in this principle inherit property of high speed and low power consumption. But this property makes overflow detection difficult during any operation. Hence the set of moduli are chosen with a very high range R. This paper discusses the use of RNS on designing finite impulse response (FIR) filter. We also propose a bit efficient moduli set for 32 bit RNS based FIR filter. A technique for mapping the data in another space is also cited. This provides the liberty to work with floating numbers with a precision. The proposed algorithms are finally used to design a FIR filter and its performance is studied.

Index Terms— Chinese remainder theorem, group delay, homomorphic mapping, moduli selection, residue number system, transpose form FIR filter.

I. INTRODUCTION

A transposed form of FIR filter is preferred always when larger filters are used. For an 8-tap, 16 bit FIR filter the device utilization and performance obtained are almost identical for both direct form as well as transposed form. But when large filters are deployed across multiple devices, the traditional approach provides a lethargic response as the input to output latency is reduced. Above this, in the transpose form if the coefficients are taken such that they are in powers of two or close to powers of two, multipliers can be eliminated by add and shift methods [1, 2, 3]. Hereby, the filter designed is a transpose form FIR filter. This allows parallel processing of the input signal and hence, the sample rate is increased. The filter is designed to operate on RNS. RNS arithmetic is carry free and each modulo arithmetic operation is independent to each other. Thus overflow detection is difficult. If a moduli set is chosen such that the operations in practical situations does not cross a bound then overflow can be avoided. Then selecting a suitable operating range Z, the signal can be mapped. This allows representation of floating point numbers into integers, with a precision, and hence RNS operation becomes easier. On the output the numbers can be converted back to decimal numbers. Since residue number representation of any integer number is unique hence the corresponding decimal number can be converted back. However there exists an error that occurred during the mapping with precision. This error propagates through the RNS operations to produce an error at the output. Although this error is tolerable, it is advisable to eliminate it at the output with some error compensation.

Fig. 1 A N-tap FIR filter in direct form transposed

II. BACKGROUND MATERIAL

A. Chinese Remainder Theorem

Consider two positive numbers, x (the dividend) and y (the divisor). Then x modulo y (abbreviated as a(mod n)) can be thought of as the remainder, on division of x by y. Now two simultaneous congruencies n=n_1(mod m_1) and n=n_2(mod m_2) are only solvable when n_1=n_2(mod gcd(m_1,m_2)). The solution is unique when m_1 and m_2 are co-prime and their gcd is 1. The Chinese Remainder Theorem (CRT) may be stated as one of the most important fundamental results in the theory of RNS.
B. RNS Basics

Let \( x_i \equiv x \mod p_i \) denote remainder of \( x \) when divided by \( p_i, i=1, 2, 3 \ldots \). Given a set of co-prime numbers \( P = \{ p_1, p_2, p_3, \ldots, p_n \} \), then every integer \( X \) in range
\[
R = \prod_{i=0}^{n-1} p_i
\]
will have unique representation \( \{ x_1, x_2, x_3, \ldots, x_n \} \). The rules for their one to one assignment given by CRT are called RNS.

C. FIR Filter

An \( N \) tap FIR filter in direct form transposed, also termed as broadcast form, is depicted in figure 1. The form can be better mapped to hardware for their one to one assignment given by CRT are called RNS.

\[
\text{y}(n) = \sum_{k=0}^{n-1} x(k) h(N-k)
\]

(1)

Where \( x \) corresponds to the input and \( a \) corresponds to the filter coefficients and \( N \) is the order of the filter. Now the coefficients \( a_k \) determine the characteristic of the filter viz. low pass, high pass, band pass, band reject etc. There are several techniques to determine the coefficients [4]. In this paper we have used hamming window technique to determine the coefficients of the FIR filter.

III. MODULI SELECTION AND MAPPING

The moduli selection is one of the crucial tasks towards a filter design based on RNS. The choice of the moduli decides the hardware complexity and speed of the system employing residue numbers [5, 6, 7].

A. Moduli Selection

**Definition:** \( N_1 \) and \( N_2 \) are called consecutive co-prime numbers if there exists no other number \( N \) such that \( N_i > N > N_2 \) which is prime to \( N_1 \).

**Theorem:** Let \( N_1, N_2, N_3, \ldots, N_k \) be a set of \( k \) consecutive co-prime numbers. Let these numbers be expressed as
\[ N_i = N_{i-1} - m_{i-1}, \]
for \( i=1, 2, 3 \ldots, k \), where \( m_{i-1} > m_{i-2} > m_{i-3} > \ldots > m_k > 0 \). Let \( N_{k+1} \) be another number that can be added to the set of co-prime numbers, i.e. \( N_{k+1} = N_{k-1} - m_k \) then \( N_{k+1} \) will be co-prime if \( \text{gcd}(N_i, m_k-m_{i-1}) = 1 \), for \( i=1, 2, 3 \ldots, k \).

**Proof:** For \( N_{k+1} \) to be co-prime to every other numbers in the set \( N_1, N_2, \ldots, N_k \),
\[
\text{gcd}(N_i, N_{k+1}) = 1
\]
or,
\[
\text{gcd}(N_{k+1}, N_i - N_{k+1}) = 1
\]
or,
\[
\text{gcd}(N_{k+1}, m_k - m_{i-1}) = 1
\]

In order to improve dynamic range of RNS with high bit efficiency, \( N_1 \) must be selected as \( 2^{m-1} \), which will be a \( m \) bit number. Then by the above method one can generate the set of co-prime numbers and use them as moduli set for RNS. Consider moduli set \( P \) of four co-prime numbers where,
\[
p_i < 2^{b_n}
\]
(2)

A 32-bit number can be represented in the RNS as four 8-bit numbers with a range 0 to \((P_r - 1)\). The numbers which cannot be represented by this scheme of representation,
\[
e = 2^b - \prod_{i=0}^{n-1} p_i
\]
(3)

Where,
- \( n \) no. of moduli, and
- \( b \) no. of bit on which filter works,
- \( n \) and \( b \) should be chosen such that \( b \) is exactly divisible by \( n \), i.e. \( b/n = 0 \).
Thus the utility factor,
\[
U_f = \frac{e}{2^b}
\]
(4)

Now choosing four co-prime numbers with \( b=32 \) and \( n=4 \), i.e., close to \( 2^8 \), gives us \( P = \{255, 254, 253, 251\} \). Hence the \( P = \{255, 254, 253, 251\} \). Since there cannot be any other combination of co-prime numbers close to \( 2^8 \), hence this is the most bit efficient moduli set with a dynamic range
\[
R = \prod_{i=0}^{n-1} p_i
\]
(5)

This makes the filter design bit efficient than any other RNS based filter.

In literature there are standard moduli set for medium dynamic ranges (less than 22 bits), three moduli set \( \{2^8, 2^9, 2^{10}\} \), and for large dynamic range (greater than 22 bits), general moduli set in form \( \{2^n, 2^{n+1}, 2^{2n+1}, \ldots\} \) with length greater than three are more efficient [5, 6]. But this moduli set shows a better bit utilization.

The dynamic range of 18 bits is supposed to be adequate for most practical situation as the bound of 18 bits is the worst outcome possible [8]. Hereby the dynamic range of \( R = 4113089310 \) is a huge margin to avoid overflow. The calculated utility factor, \( U_f = 0.04235 \). Lower the utility factor better is the moduli set.

B. Mapping

The next step is to map the number i.e. to convert the floating point numbers to integers. The operating range is suitably taken such that no overflow occurs.
Mapping is a special correspondence between the members (elements) of two fields. Two homomorphic systems have the same basic structure. Their elements and operations appear different; results on one system often apply as well to the other system [9]. Operating on floating numbers is a difficult task in residue arithmetic. Hence we convert the floating point number to an integer in a range ±Z. The mapping is done such that the operating range $Z < R$.

Precisely, if

$$Z \leq R$$

then overflow of any operation can be easily avoided. If the floating point number $x$ is in range $\pm A$, and $A < Z$, then

$$P_{sn} = \frac{Z}{A}$$

$$X = \frac{x}{P_{sn}}$$

In this mapping the range $\pm A$ has predominant effect on the precision with which the technique can handle the floating point numbers. As value of $A$ increases the precision decreases and the floating point numbers accuracy deteriorates since precision decrease.

IV. RNS FIR FILTER DESIGN

The coefficients of the filter decide the nature of the filter. There are several techniques in literature that generates the coefficients of a filter for a particular behavior. In RNS, since operations are split into smaller parallel operations which are independent of each other, there exists $n$ number of filters operating simultaneously. This provides very high speed architecture. Figure 2 and figure 3 show the RNS based filter and expansion of mod $p_i$ filter block used in the block diagram of the original filter design in figure 2. The residue numbers and converted back by the Chinese remainder theorem based reverse conversion method [10].

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A. Coefficients:

The filter coefficients are determined using a Hamming-window based, linear-phase filter with normalized cutoff frequency $0.3\pi$ rads. By default the filter is normalized so that
the magnitude response of the filter at the center frequency of the pass band is 0 dB. The magnitude response is compared with the original response to find out the error. The phase response figure 4(b) is linear over the filter pass band, but it loses its piecewise linearity after the cut-off region, unlike the original FIR filter. This causes a group delay as given in the figure 4(c).

B. Filter Performance

Group delay is a measure of time distortion which is calculated by differentiating the phase response versus frequency. It is also a measure of the slope of the phase response. The linear portion of the group delay plot represents the average signal-transit time and deviations from linear phase are transformed into deviations from constant group delay.

The variations in group delay cause signal distortion, just as deviations from linear phase cause distortion. But since the group delay occurs in the high frequency region of the signal, any distortion in that part is of least botheration. Thus the filter even being a non-linear filter characterizes itself as an efficient filter.

C. Performance Analysis

The performance analysis is done with respect to the generic traditional FIR filter [11]. However the phase response shows deviation as because there are few data as well as coefficients those are truncated to their higher value during the homomorphic mapping and RNS conversion.

This creates an error which propagates through the filter to its output. The poles zero diagram, in figure 7, of the RNS based filter shows the filter stability. The pole zero plot is generally used to analyze the stability of the system, as we can observe in this case the designed filter. The poles and the zeros of the RNS filter designed are almost same as the general FIR filter.

The stability of the designed filter is not affected at all by the proposed design methodology.
V. RESULTS

The impulse response, step response, magnitude response and the pole zero plot analyses the filter designed with the coefficients obtained from hamming window technique and implemented on residue arithmetic with a homomorphic mapping. The results are satisfactory with the moduli set proposed. However the group delay was quite high in the stop band, which of course does not affect the output.

VI. CONCLUSION

The RNS based filter design can produce very high speed FIR filter. A FIR filter based on the residue number system is exhibited and the filter performance is analyzed extensively. The new proposed lowpass FIR filter operates over a high range and the homomorphic mapping provide the incorporation of fractional part of the signal with a calculated precision. Hereby the bit efficiency of the residue number based FIR filter is concluded to have a satisfactory performance. However, the major problem with the RNS based filter is the detection of overflow. If the overflow detection can be done without any use of lookup tables unlike many methods cited in literature viz. [12, 13], then these filters can be optimized to be highly accurate filter which will inherit the property of high speed and low power consumption.

REFERENCES


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