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Performance Evaluation of Equalizers and Different Diversity Techniques using OFDM

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Abstract- **In this paper, we analyzed the performance of different diversity techniques, which can be easily applied with orthogonal frequency division multiplexing (OFDM) systems with different equalizing methods. Orthogonal frequency division multiplexing is well known to be a useful technique for high rate data transmission over a frequency selective fading channel. However, for coherent detection of OFDM signals, fading compensation techniques are required to mitigate amplitude and phase distortions due to the multi-path channel fading. The fading compensation technique becomes even more crucial when QAM is used for OFDM systems. A channel estimation technique using periodically inserted pilot symbols in the data stream is well known to provide a reliable way to mitigate the distortions.**

I. INTRODUCTION

We consider the iterative channel estimation and decoding algorithms for QAM modulated orthogonal OFDM signals. A loss of sub-channel orthogonality due to time-variant multipath channels in orthogonal frequency-division multiplexing systems leads to ICI which increases the error floor in proportion to the Doppler frequency. In this paper, we use two equalization techniques LSE & ZF which can compensate for the effect of ICI in a multipath fading channel is used. In these techniques, the equalization of the received OFDM signal is achieved by using the assumption that the channel impulse response (CIR) varies in a linear fashion during a block period and by compensating for the ICI terms that significantly affect the bit-error rate (BER) performance. We are using two Diversity techniques (Maximal Ratio Combining and Equal Gain Combining) in multi path fading environment along with the additive white Gaussian noise.

OFDM is well known to be a useful technique for high rate data transmission over a frequency selective fading channel. However, for coherent detection of OFDM signals, fading compensation techniques are required to mitigate amplitude and phase distortions due to the multi-path channel fading. The fading compensation technique becomes even more crucial when *QAM* is used for OFDM systems.

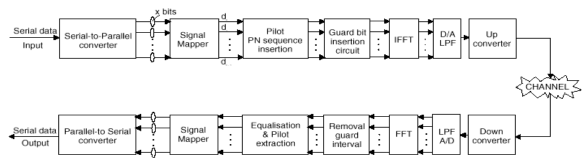

Figure 1: Block Diagram of OFDM

Least Square Error equalizer is the preferred choice in equalizers to compensate for ISI and to provide the optimum solution.

The ZF Equalizer applies the inverse of the channel to the received signal, to restore the signal before the channel. It has many useful applications. The name Zero Forcing corresponds to bringing down the ISI to zero in a noise free case. This will be useful when ISI is significant compared to noise.

Several recent works have investigated the performance of MRC diversity with a single transmit antenna and multiple receive antennas in the presence of Co-Channel Interference. The results in assume that the powers associated with all received signals are the same or are completely different, which precludes the situation in cellular systems with power control where in-cell interferers all have the same received power while out-of-cell interference signals have different powers.

EGC is an important diversity technique that is often used to mitigate fading in various wireless communications systems. The EGC has several practical advantages over other diversity techniques, because it has close to optimal performance and yet is simple to implement. The OP is the primary performances measure for all diversity systems, particularly those exposed to CCI such as the cellular mobile systems.

II. PRINCIPLE OF THE IFFT OPERATOR

The FFT is the Fast Fourier Transform operator. This is a matrix computation that allows the discrete Fourier transform to be computed (while respecting certain conditions). The FFT works for any number of points. The operation is simpler when applied for a number *N* which is a power of 2(e.g. *N* 256). The IFFT is the Inverse Fast Fourier Transform operator and realizes the reverse operation shown in below figure 2.

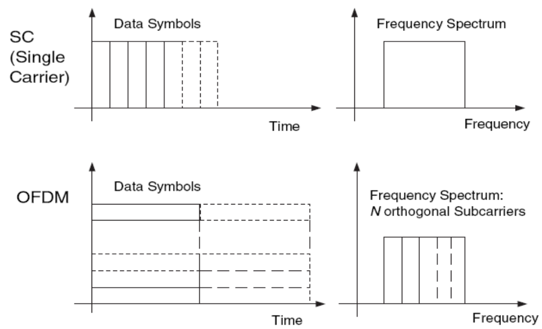

Figure 2: Time and frequency representation of the SC and OFDM. In OFDM, N data symbols are transmitted simultaneously on N orthogonal sub-carriers.

OFDM theory shows that the IFFT of magnitude N, applied on N symbols, realizes an OFDM signal, where each symbol is transmitted on one of the N orthogonal frequencies. The symbols are the data symbols of the type BPSK, QPSK, QAM-16 and QAM-64 introduced in the previous section. Figure 3 shows an illustration of the simplified principle of the generation of an OFDM signal. In fact, generation of this signal includes more details that are not shown here for the sake of simplicity.

If the duration of one transmitted modulation data symbol is T_d, then $T_d = 1/\Delta f$, where Δf is the frequency bandwidth of the orthogonal frequencies. As the modulation symbols are transmitted simultaneously,

T_d = duration of one OFDM symbol

= duration of one transmitted modulation data symbol.

This duration, Δf, the frequency distance between the maximums of two adjacent OFDM sub-carriers, can be seen in Figure 3. This figure shows how the neighboring OFDM sub-carriers have values equal to zero at a given OFDM sub-carrier maximum, which is why they are considered to be orthogonal. In fact, duration of the real OFDM symbol is a little greater due to the addition of the Cyclic Prefix (CP).

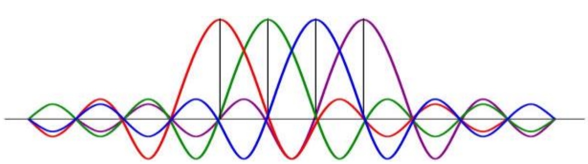

Figure 3: Presentation of the OFDM sub-carrier frequency.

III. Time Domain OFDM Considerations

After application of the IFFT, the OFDM theory requires that a Cyclic Prefix (CP) must be added at the beginning of the OFDM symbol (see Figure 3). Without getting into mathematical details of OFDM, it can be said that the CP allows the receiver to absorb much more efficiently the delay spread due to the multi-path and to maintain frequency orthogonality. The CP that occupies a duration called the Guard Time (GT), often denoted T_G, is a temporal redundancy that must be taken into account in data rate computations. The ratio T_G/T_d is very often denoted G in WiMAX/802.16 documents.

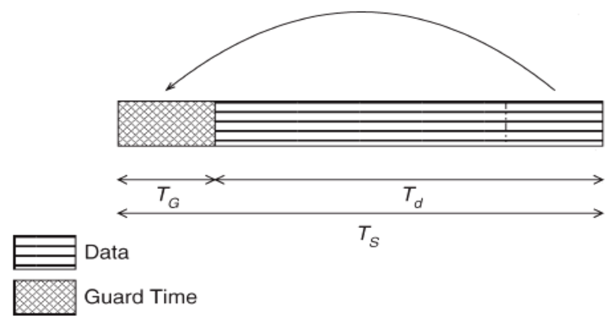

Figure 4: Cyclic Prefix insertion in an OFDM symbol.

The choice of G is made according to the following considerations: if the multi-path effect is important (a bad radio channel), a high value of G is needed, which increases the redundancy and then decreases the useful data rate; if the multi-path effect is lighter (a good radio channel), a relatively smaller value of G can be used. For OFDM and OFDMA PHY layers, 802.16 defined the following values for G: 1/4, 1/8, 1/16 and 1/32. For the mobile (OFDMA) WiMAX profiles presently defined, only the value 1/8 is mandatory.

IV. USE OF CYCLIC PREFIX IN MULTIPATH CHANNEL

Cyclic prefix acts as a buffer region where delayed information from the previous symbols can get stored.

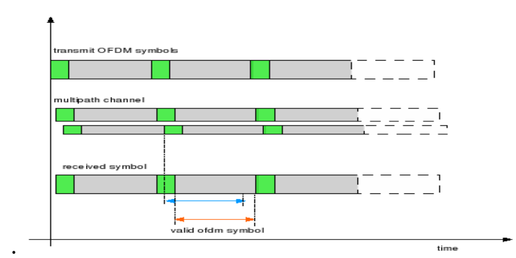

Figure 5: OFDM symbol with multipath

The receiver has to exclude samples from the cyclic prefix which got corrupted by the previous symbol when choosing the samples for an OFDM symbol. Further, from the previous section, we learned that a sinusoidal added with a delayed version of the same sinusoidal does not affect the frequency of the sinusoidal (it only affects the amplitude and phase).

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V. FUNCTIONAL DESCRIPTION OF OFDM MODULATION AND DEMODULATION

This design example has two parts: OFDM modulation, which includes IFFT and cyclic prefix insertion with bit reversal, and OFDM demodulation, which includes cyclic prefix removal and data buffer for rate change. Figure 6 shows the top-level integration of the two parts.

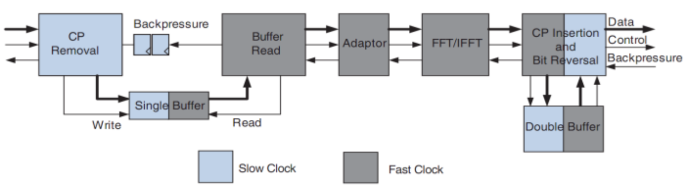

Figure 6: Structure for OFDM Modulation and Demodulation Design Example

VI. ADDITIVE WHITE GAUSSIAN NOISE CHANNEL

AWGN is a channel model in which the only impairment to communication is a linear addition of wideband or white noise with a constant spectral density (expressed as watts per hertz of bandwidth) and a Gaussian distribution of amplitude. The model does not account for fading, frequency selectivity, interference, nonlinearity or dispersion. However, it produces simple and tractable mathematical models which are useful for gaining insight into the underlying behavior of a system before these other phenomena are considered.

Channel Capacity - The AWGN channel is represented by a series of outputs Y_i at discrete time event index *i*. Y_i is the sum of the input X_i and noise, Z_i, where Z_i is independent and identically-distributed and drawn from a zero-mean normal distribution with variance *n* (the noise). The Z_i are further assumed to not be correlated with the X_i.

$$Z_i \sim N(0,n) \tag{1}$$

$$Y_i = X_i + Z_i. \tag{2}$$

The capacity of the channel is infinite unless the noise n is nonzero, and the X_i are sufficiently constrained. The most common constraint on the input is the so-called "power" constraint, requiring that for a codeword $(x_1,x_2,...,x_n)$ transmitted through the channel, we have:

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 \leq P \tag{3}$$

where *P* represents the maximum channel power. Therefore, the channel capacity for the power-constrained channel is given by:

$$C = \max_{f(x) s.t. E(X^2) \leq P} I(X;Y) \tag{4}$$

Where *f*(*x*) is the distribution of *X*. Expand *I*(*X*;*Y*), writing it in terms of the differential entropy:

$$I(X;Y) = h(Y) - h(Y|X) = h(Y) - h(X+Z|X) \ = h(Y) - h(Z|X) \tag{5}$$

But *X* and *Z* are independent, therefore:

$$I(X;Y) = h(Y) - h(Z) \tag{6}$$

Evaluating the differential entropy of a Gaussian gives:

$$h(Z) = \frac{1}{2}\log(2\pi en) \tag{7}$$

Because *X* and *Z* are independent and their sum gives *Y*:

$$E(Y^2) = E(X+Z)^2 = E(X^2) + 2E(X)E(Z) + E(Z^2) = P + n \tag{8}$$

From this bound, we infer from a property of the differential entropy that

$$h(Y) \leq \frac{1}{2}\log(2\pi e(P+n)) \tag{9}$$

Therefore the channel capacity is given by the highest achievable bound on the mutual information:

$$I(X;Y) \leq \frac{1}{2}\log(2\pi e(P+n)) - \frac{1}{2}\log(2\pi en) \tag{10}$$

Where *I*(*X*;*Y*) is maximized when:

$$X \sim N(0,P) \tag{11}$$

Thus the channel capacity *C* for the AWGN channel is given by:

$$C = \frac{1}{2}\log(1 + \frac{P}{n}) \tag{12}$$

VII. ACHIEVABILITY

A codebook, known to both encoder and decoder, is generated by selecting codewords of length n, i.i.d. Gaussian with variance $P - \varepsilon$ and mean zero. For large n, the empirical variance of the codebook will be very close to the variance of its distribution, thereby avoiding violation of the power constraint probabilistically.

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Received messages are decoded to a message in the codebook which is uniquely jointly typical. If there is no such message or if the power constraint is violated, a decoding error is declared.

Let $X^n(i)$ denote the codeword for message i, while Y^n is, as before the received vector. Define the following three events:

1. Event U:the power of the received message is larger than P.
2. Event V: the transmitted and received codewords are not jointly typical.
3. Event E_j: $(X^n(j),Y^n)$ is in $A_\epsilon^{(n)}$, the typical set where $i \neq j$, which is to say that the incorrect codeword is jointly typical with the received vector.

An error therefore occurs if U, V or any of the E_i occur. By the law of large numbers, $P(U)$ goes to zero as n approaches infinity, and by the joint Asymptotic Equipartition Property the same applies to $P(V)$. Therefore, for a sufficiently large n, both $P(U)$ and $P(V)$ are each less than ε. Since $X^n(i)$ and $X^n(j)$ are independent for $i \neq j$, we have that $X^n(i)$ and Y^n are also independent. Therefore, by the joint AEP, $P(E_j) = 2^{-n(I(X;Y)-3\epsilon)}$. This allows us to calculate $P_e^{(n)}$, the probability of error as follows:

$$\begin{aligned} P_e^{(n)} &\le P(U) + P(V) + \sum_{j\neq i} P(E_j) \\ &\le \epsilon + \epsilon + \sum_{j\neq i} 2^{-n(I(X;Y)-3\epsilon)} \\ &\le 2\epsilon + (2^{nR}-1)2^{-n(I(X;Y)-3\epsilon)} \\ &\le 2\epsilon + (2^{3n\epsilon})2^{-n(I(X;Y)-R)} \\ &\le 3\epsilon \end{aligned} \quad (13)$$

Therefore, as n approaches infinity, $P_e^{(n)}$ goes to zero and $R < I(X;Y) - 3\epsilon$. Therefore there is a code of rate R arbitrarily close to the capacity derived earlier.

Coding Theorem Converse :

Here we show that rates above the capacity are not achievable. $C = \frac{1}{2}\log(1+\frac{P}{n})$

Suppose that the power constraint is satisfied for a codebook, and further suppose that the messages follow a uniform distribution. Let W be the input messages and $\hat{W}$ the output messages. Thus the information flows as:

$$W \longrightarrow X^{(n)}(W) \longrightarrow Y^{(n)} \longrightarrow \hat{W} \quad (14)$$

Making use of Fano's inequality gives:

$$H(W|\hat{W}) \le 1 + nRP_e^{(n)} = n\epsilon_n \quad (15)$$

where $\epsilon_n \rightarrow 0$ as

$$P_e^{(n)} \rightarrow 0 \quad (16)$$

Let X_i be the encoded message of codeword index i. Then:

$$\begin{aligned} nR = H(W) &= I(W;\hat{W}) + H(W|\hat{W}) \\ &\le I(W;\hat{W}) + n\epsilon_n \\ &\le I(X^{(n)};Y^{(n)}) + n\epsilon_n \\ &= h(Y^{(n)}) - h(Y^{(n)}|X^{(n)}) + n\epsilon_n \\ &= h(Y^{(n)}) - h(Z^{(n)}) + n\epsilon_n \\ &\le \sum_{i=1}^{n} Y_i - h(Z^{(n)}) + n\epsilon_n \\ &\le \sum_{i=1}^{n} I(X_i;Y_i) + n\epsilon_n \end{aligned} \quad (17)$$

Let P_i be the average power of the codeword of index i:

$$P_i = \frac{1}{2^{nR}}\sum_w x_i^2(w) \quad (18)$$

Where the sum is over all input messages w. X_i and Z_i are independent, thus the expectation of the power of Y_i is, for noise level N:

$$E(Y_i^2) = P_i + N \quad (19)$$

And, if Y_i is normally distributed, we have that

$$h(Y_i) \le \frac{1}{2}\log 2\pi e(P_i + N) \quad (20)$$

Therefore, We may apply Jensen's equality to $\log(1 + x)$, a concave (downward) function of x, to get:

Because each codeword individually satisfies the power constraint, the average also satisfies the power constraint. Therefore

$$\frac{1}{n}\sum_{i=1}^{n}\frac{P_i}{N} \quad (21)$$

Which we may apply to simplify the inequality above and get:

$$\frac{1}{2}\log\left(1+\frac{1}{n}\sum_{i=1}^{n}\frac{P_i}{N}\right) \le \frac{1}{2}\log\left(1+\frac{P}{N}\right) \quad (22)$$

Therefore, it must be that

$$R \le \frac{1}{2}\log\left(1+\frac{P}{N}\right) + \epsilon_n. \quad (23)$$

Therefore, R must be less than a value arbitrarily close to the capacity derived earlier, as $\epsilon_n \rightarrow 0$.

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$$nR \leq \sum(h(Y_i) - h(Z_i)) + n\epsilon_n$$
$$\leq \sum\left(\frac{1}{2}\log(2\pi e(P_i+N)) - \frac{1}{2}\log(2\pi eN)\right) + n\epsilon_n$$
$$\frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}\log\left(1+\frac{P_i}{N}\right) \leq \frac{1}{2}\log\left(1+\frac{1}{n}\sum_{i=1}^{n}\frac{P_i}{N}\right)$$
$$= \sum\frac{1}{2}\log(1+\frac{P_i}{N}) + n\epsilon_n \quad (24)$$

VIII. RESULTS AND DISCUSSION

In digital modulation techniques a set of basic functions are chosen for a particular modulation scheme. Generally the basic functions are orthogonal to each other. Basis functions can be derived using 'Gram Schmidt orthogonalization procedure. Once the basic functions are chosen, any vector in the signal space can be represented as a linear combination of the basic functions.

BER Model of Zero Forcing Equalizer (ZF)

The attached Mat lab simulation script performs the following:

(a) Generation of random binary sequence.

(b) Convolving the symbols with a 3-tap fixed fading channel.

(c) Adding White Gaussian Noise.

(d) Computing the equalization filter at the receiver.

(e) Demodulation and conversion to bits.

(f) Counting the number of bit errors.

(g) Repeating for multiple values of Eb/No.

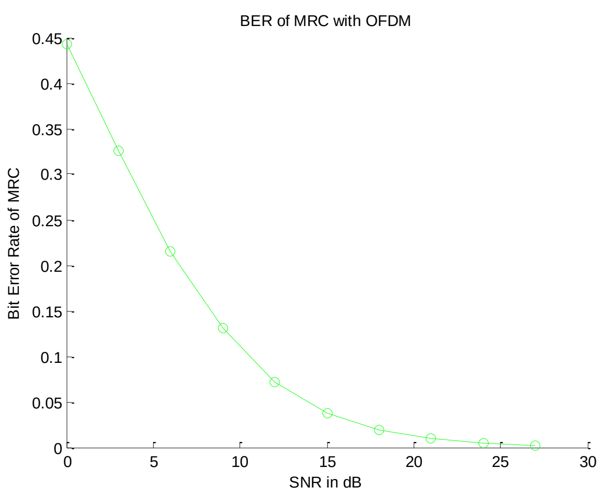

Figure 7: BER plot of Maximal Ratio Combining

Simulating BER with Equal Gain Combining (EGC):
The Matlab script performs the following

(a) Generate random binary sequences.

(b) Multiply the symbols with the channel and then add white Gaussian noise.

(c) Chose that receive path, equalize the received symbols per maximal ratio combining.

(d) Perform hard decision decoding and count the bit errors.

(e) Repeat for multiple values of $\frac{E_b}{N_0}$and plot the simulation and theoretical results.

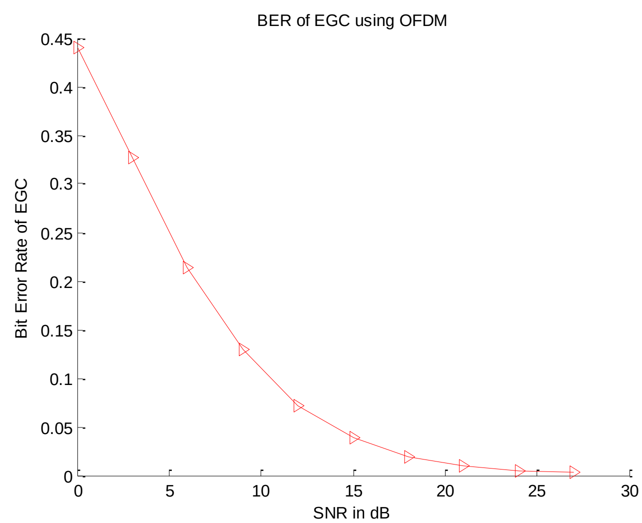

Figure 8: BER plot of Equal Gain Combining

Look at the other situation with 32-QAM and 1024 subchannels and 1000 iterations we find that no change in performance of MRC. Therefore MRC has won race in OFDM based receivers in our case.

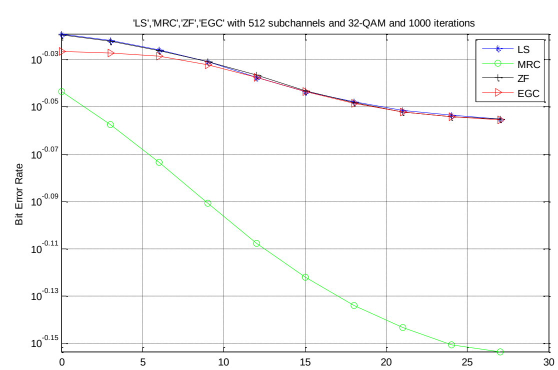

Figure 9: BER with LS, MRC, ZF & EGC with 1024 subchannels and 32-QAM with 1000 iterations

IX. CONCLUSION

In this paper, bandwidth efficient channel identification algorithm utilizing receiver diversity is proposed. In the noiseless case, the algorithm can perfectly retrieve the channels up to a scalar factor. In the presence of noise, the algorithm has very low complexity. If we look at the simulation result with 1024 subchannels and 1000 iterations, we find that the BER curve is becoming linear and therefore with high number of subcarriers and with 1000 iterations we get more better results compared to 256 and 512 subchannels conditions.

One thing is clear here that with lower order QAM modulation techniques results are not much comparative, but results in bunching like the optimum performers. Only the change between performances can be seen with lower number of subcarriers and with 1000 iterations.

Therefore we conclude with this assumption that with 1000 iterations we have better performance of used algorithms and MRC is very much able to show expected results.

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