Abstract— This paper presents the modelling of negative permittivity metamaterials structures based on perfectly conducting wires using the transmission-line modelling (TLM) method. First, an array of straight parallel conducting metallic thin-wires is modeled in a TLM lattice with the symmetrical condensed node (SCN). Secondly, an S-shaped thin-wire structure is studied and analyzed to control the plasma frequency. Both structures were modeled using the interface-wire-SCN (IW-SCN) approach. The transmission and reflection coefficients obtained by this approach and by the commercial simulator CST Microwave Studio are in good agreement. In addition, the retrieved effective materials permittivities are computed to show the electromagnetic properties of the studied wire structures.

Index Terms— Metamaterials, Wire medium, TLM, Plasma frequency.

I. INTRODUCTION

Commercial tools based on numerical techniques are broadly used in the resolution of electromagnetic problems. For this purpose, there are several commercial packages formulated both in time and in frequency domain. However, they do not permit the accuracy and versatility needed in some specific applications such as in modelling metamaterials. Hence, it is important to have versatile tools based on electromagnetic approaches which can be manipulated and used optimally to deal with metamaterials. In our case, we have used the TLM method [1] to build an efficient tool to model metamaterials, especially, those based on cylindrical conducting thin-wires considered in this paper.

Metamaterials are inherently artificial materials, which are not found in nature, and which yield interesting electromagnetic responses. Since the initial theoretical study by Veselago [2] and the first experimental verification by J. B. Pendry et al. [3-4], the interest in metamaterials continues to grow and many new applications were suggested. Since the metamaterials structures depend on their parameters we can find many types: single negative medium, where the permittivity or permeability is negative; and double negative medium where both the permittivity and permeability are negative. One possible design of a material with negative permittivity is the use of a three-dimensional lattice formed by thin parallel conducting wires. This structure is known within the microwave community for a long time as an artificial dielectric, or wire medium. In the recent literature, wire media have received increasing attention owing to their potential new applications, such as antenna reflectors and high gain compact antennas [5-6]. To achieve negative permeability a periodic combination of split-ring resonators (SRR) was suggested. A double negative medium is obtained by alternating periodically wires and SRRs [7].

There are many known numerical techniques and enough worked out theories describing electromagnetic properties of wire metamaterials structures. For example, in [8-10] the propagation of electromagnetic waves through arrays of perfectly conducting cylindrical thin-wires for both fundamental polarizations is studied and some results were compared with those obtained by commercial tools [11-12]. Modelling of metamaterials by means of the TLM method is a difficult task and their treatment in the literature is relatively scarce in spite of their importance [13]. Furthermore, the study of propagation through metamaterial structures based on wires using the TLM method with wire nodes has not been done yet to our best knowledge. In this paper, we simulate and analyze a design of S-shaped wire structures with negative permittivity which represents an alternative to the classical wire media [14]. The TLM model used is based on the implementation of wires in the interface of the famous symmetrical condensed node (SCN). The simplicity and particular structure of the SCN with interface wire model (IW-SCN) permits the study and the design of many different structures based on conducting thin-wires. In reference [15], the description and full details of the IW-SCN algorithm are given.
The numerical results obtained by the IW-SCN approach and the commercial tool CST MWS give the transmission and reflection parameters which are used to extract the effective permittivity of metamaterial structures studied in this paper by means of the retrieval method proposed by Smith [16] based on effective medium theory [17-18]. For all the metamaterial structures considered, the obtained results are in good agreement.

II. DESCRIPTION OF THE MODELLED STRUCTURES

Fig. 1. (a) 3x3 array of perfectly conducting wires embedded in air. The wires radius is \( r \), their periodicity along \( x \) and \( y \) is \( a \), and they are directed along \( z \) axis. (b) The proposed S-shaped wire structure. The wires along \( x \) have the same length \( \ell_x \) and the ones along \( z \) have the same height \( \ell_z \). The centered straight wire is at the half period between the two S units and has a height of \( 2\ell_z \).

The first wire metamaterial structure is formed by a 3x3 periodic array of cylindrical perfectly conducting thin-wires (Fig. 1(a)). This structure is well known in the literature [3-4]. The wire radius is \( r \), the length is \( 2\ell_z \) and the periodicity is \( a \). A plane wave polarized Gaussian pulse, parallel to the wires, is incident upon the wire structure.

To validate the results obtained by IW-SCN, simulations with CST Microwave Studio were performed for which this structure may be considered as a parallel plate waveguide.

The second configuration is the S-shaped wire structure as shown in Fig. 1(b), where perfectly conducting wires are perpendicularly interconnected forming a two squared S separated by a thin wire as illustrated in the same figure. This structure yields to an eightlike pattern when viewed from the left or right side [14]. Contrary to the conventional wire medium for which the length of wires does not affect the transmission and reflection coefficients, the wires length in the S-shaped wire structure has an important effect on those coefficients, and thus on parameters like permittivity and permeability. To ensure the resonance we insert a single wire at the middle between the two S-shaped units. To perform the modelling of the S-shaped structure using the TLM method with wires in the interface of the SCN, a Gaussian plane wave polarized along \( z \)-direction and propagated along \( y \)-direction is used.

III. RESULTS AND DISCUSSIONS

Fig. 2. The reflection coefficient results for the 3x3 wire structure of Fig.1 (a).

We begin this analysis by applying the IW-SCN approach to a 3x3 wire array of Fig. 1(a) placed in air as a host medium. The TLM workspace is \((20 \times 20 \times 10) \times \Delta \ell\), where \( \Delta \ell \) is the SCN step size taken to be \( \Delta \ell = 5 \) mm. The length of each wire is taken to be \( 2\ell_z = 20 \) mm, the wire radius is \( r = 0.5 \) mm and the wires periodicity along \( x \)-direction and \( y \)-direction is \( a = 5 \) mm. In Fig. 2, we plot the reflection coefficient for this structure obtained by the TLM method with wires in the interface of the SCN, a Gaussian plane wave polarized along \( z \)-direction and propagated along \( y \)-direction is used.

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Thicker wires undergo higher resistive losses at microwave frequencies which tend to increase the plasma frequency. For left handed media, it is desired to use thick wires to include the entire frequency range of double negative operation. If a low plasma frequency is desired, then we may embed the wires in a high permittivity dielectric medium. This method may be necessary when trying to design zero-index metamaterial structures at X-band or lower frequency bands. The dielectric substrate may be used to lower the resonant plasma frequency without altering the inclusion structure.

Another possible way to reduce the plasma frequency is to modify the geometry of the structure by varying the wires length and keeping the radius and the periodicity constants, as is the case with the S-shaped structure of Fig. 1(b). The dimensions of the structure are as the following: the wires radius is $r = 0.5$ mm, the distance between the two S-units is taken to be $d = 10$ mm and the wires length along $x$ and $z$ directions are, respectively, $\ell_x$ and $\ell_z$ (i.e. $\ell_x = \ell_z$).

Using the TLM method, the simulation of this structure has been carried out considering a coarse regular mesh with the same dimensions as the first structure and the length of the centered wire is $2\ell_x = 4\Delta l = 20$ mm. Fig. 3 shows the reflection coefficient obtained from modelling this structure using TLM with IW-SCN and CST. Two first resonances are visibly separated by a stop band where evanescent modes dominate. The first resonance at 4 GHz corresponds to the permittivity ($\varepsilon$) becoming negative and the second one is the plasma resonance which indicates the frequency range where the permittivity approaches zero. The location of the plasma frequency where evanescent modes no longer dominate is at 10.3 GHz for results obtained using the TLM with IW-SCN and at 10.5 GHz using CST simulations. The agreement between IW-SCN approach and CST simulations is excellent.

Fig. 3. The reflection coefficient results for the S-shaped structure of Fig.1 (b).

Fig. 4. Reflection coefficient using IW-SCN for different configurations of the S-shaped structure of Fig.1 (b). The values between parenthesis represent the pair $(\ell_x, \ell_z)$ for a constant wires radius $r = 0.5$ mm and periodicity $d = 10$ mm.

Fig. 5. The effective permittivity of the structure of Fig. 1 (a). Continuous line: real part, dashed line: imaginary part.

Fig. 6. The effective permittivity of the structure of Fig.1 (b). $(\ell_x, \ell_z) = (5$ mm, 10 mm), $r = 0.5$ mm, and $d = 10$ mm. Continuous line: real part, dashed line: imaginary part.
By keeping the wire radius \( r = 0.5 \) mm and the periodicity \( d = 10 \) mm unchanged, we plot in Fig. 4 the reflection coefficient using IW-SCN model for different configurations by varying the pair \((f_1, f_2)\) of the wire lengths along x- and z-directions. As in Fig. 3, the first resonance corresponds to the permittivity becoming negative and the second one gives the plasma frequency for different values of the pair of lengths \((f_1, f_2)\). It is clearly seen that the length of wires in this structure is a decisive parameter when computing the plasma frequency. As can be seen, the plasma frequency decreases when the wires length increases. With this technique we can configure the S-shaped structure to obtain the desired range of frequency for which the evanescent waves no longer dominate. In figures 5 and 6 we plot the corresponding relative permittivity obtained by using the retrieval method for both the 3x3 wires structure and the S-shaped wires structure respectively. The results of the S-parameters from which the permittivities are derived have been obtained using the IW-SCN approach.

IV. CONCLUSION

In this paper we have shown that the TLM method with the interface-wire-symmetrical condensed node (IW-SCN) can be applied to the modelling of metamaterials based on conducting wires. Taking advantage of IW-SCN model, we have proposed a viable alternative to the classical wire media to control the plasma frequency which allows the control of the frequency range of the negative permittivity. The basic concept embodied in the new S-shaped structure could be extended to multidimensional media. Furthermore, the simplicity of the IW-SCN model makes it suitable for other structures based on conducting thin wires.

REFERENCES