Performance Evaluation of Adaptive Algorithm for Hearing Aid Implementation

Obidike I.A, Oguejiofor O.S, Ejiofor H.C, Okechukwu G.N

Abstract—The performance of adaptive algorithm is very critical to the automatic control of the gain of the digital filter. The major centre of attraction in the improvement and implementation of digital hearing aid had been on the filter. Least mean square (LMS) algorithm had posed a challenge of not being operated in a non-stationary environment in the last decade. In effect, a Normalize Least Mean Square (NLMS) block model has been developed and simulated in Matlab. The filter output was thereafter compared and evaluated with that of the existed LMS block model. The result of the simulation showed that NLMS algorithm maintained a faster speed of convergence with a minimal mean square error when step size is varied.

Index Terms—Adaptive Algorithm, Filters, LMS, MMISE, NLMS.

I. INTRODUCTION

Hearing impairment has been defined as person’s inability to hear within the stipulated hearing frequency meant for a normal hearing person. The allowable frequency for hearing ranges from 20Hz to 20 kHz. But within this range of hearing, the ear is only sensitive to 1 kHz – 4 kHz [1]. Therefore any frequency below 1 kHz cannot be heard by the ear; while above 4 kHz can simply damage the ear. Several factors have been accrued to hearing loss. Few of these include continuous increase in the volume of electronic gadgets, frequent exposure of the ear to excessive noise from industrial machines, gun fire or explosions, old age, diseases such as nerve deafness, drugs among others can lead to conductive loss or sensory neural loss or even both [2, 3].

As a result of the increased cases of hearing impairment round the world, there has been huge efforts and successes obtained from research in curbing this menace. Nevertheless, the emergence of hearing aid brought relief and provided an alternative solution to medical surgery, as the advent of hearing aid brought relief and provided an alternative solution to medical surgery, as emergence of hearing aid brought relief and provided an alternative solution to medical surgery. Nevertheless, the emergence of hearing aid brought relief and provided an alternative solution to medical surgery. Nevertheless, the emergence of hearing aid brought relief and provided an alternative solution to medical surgery.

In the case of conventional analog type of hearing aid, it amplifies both the amplifying sound and offsetting loss [4]. In the case of alternative solution to medical surgery, as emergence of hearing aid brought relief and provided an alternative solution to medical surgery, as emergence of hearing aid brought relief and provided an alternative solution to medical surgery, as emergence of hearing aid brought relief and provided an alternative solution to medical surgery.

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II. ADAPTIVE FILTER STRUCTURE

An adaptive filter is a self-designing and time-varying system that uses a recursive algorithm continuously to adjust its tap weights for operation in an unknown environment [6]. Ideally, it uses the filter parameter of the previous signal to automatically adjust the filter parameter of the next iterated signal in order to achieve optimum weight. Basically, adaptive filter consists of two parts as shown in figure1: a digital filter for performing actual digital signal processing and an adaptive algorithm for adjusting the coefficients (the adaptive tap weights) of the filter. Therefore, adaptive filter can be said to have a self modifying property. The capability of the filter to operate satisfactorily in an unknown environment and track time variations of input statistics make the adaptive filter a powerful device for signal processing and control applications [7]. The essence of the filter structure in figure 1 is to minimize mean square error between the filter output signal, y(n), referred to as the filtered signal and the desired signal, d(n) as the targeted signal [8]. The general set up of the structure constitute an FIR digital filter block and the adaptive algorithm block, since hearing aid where these components can be applied is a time varying system. The figure 1 further showed an input signal, x(n), x(n)+s(n) indicating the input signal corrupted with noise,
adaptive filter output signal, y(n) and the reference or desired signal, d(n). The error signal e(n) is the signal obtained from the difference signal between d(n) and y(n). What happens is that after sampling of x(t) signal, the signal is converted from analog to digital signal x(n) using analog-to-digital converter (ADC). Thereafter, the digital signal proceeds for digital signal processing done by an FIR filter. The output y(n) of the filter is later compared with d(n) for error detection and minimization. Once e(n) signal is detected, the adaptive algorithm triggered for adjustment and automatic update to the filter coefficients of the next iterated input signal via a feedback mechanism. This adaptation process continues until the e(n) signal is totally minimized. At this point, the y(n) signal is said to be approximately equal to d(n) signal. Since noise cannot be totally eliminated in communication systems, one of the options is minimization. This minimization process of the mean square error has been one of the major aims of this work. The next question to be considered should be the time taken to get the error minimized. Here, the question of the convergence rate comes to play.

\[ e(n) = d(n) - y(n) \]  

\[ y(n) = \sum_{k=0}^{N-1} w_k (n)x(n-k) = w^H x(n) \]  

Where
\[ x(n) = [x(n), x(n-1), ..., x(n-k+1)]^{T} \]  

\[ e(n) = d(n) - y(n) \]  

When e(n) = 0, y(n) = d(n) .This condition is for an ideal system since error cannot be eliminated totally.

3. \[ \xi = e[d[n] - y[n]^T] \]  

% w[n] is selected to minimize the MSE

\[ (4) \]

A. **Minimum Mean Square Error**

The minimum square error has been defined by Scott .C. Douglas in [9] as a metric indicating how well a system can adapt to a given solution. A minimum MSE simply indicates how well the adaptive system has been accurately modeled, predicted, adapted to a solution for the system. Several factors that can help determine the minimum MSE include the quantization noise, the filter order, the amount of noise in the system and error of the gradient due to the finite step size.

B. **Convergence Rate**

The convergence rate is defined as the number of iterations required for the algorithm under stationary conditions to converge “close enough” to the optimum Wiener solution in the mean square sense. At this rate, the adaptive filter coefficient is said to approach its optimum value. The speed of convergence rate increases as the value of the step size increases and vice versa. The faster rate of convergence of the algorithm allows for rapid adaptation in a stationary environment of unknown statistics. The quantitative measure by which the final value of mean-square error (MSE) is averaged over an ensemble of adaptive filters deviates from the minimum MSE more severely as the rate of convergence becomes faster, which means that their trade-off problem exists. Apart from step size, \( \mu \), other parameters in which the speed of convergence of an adaptive filter depends on are the filter length L, the initial filter coefficient value, the amplitude and the correlation statistics of the signals [9].

III. **DESIGN OF AN FIR DIGITAL FILTER**

A digital hearing aid consists of an in-built microphone for sound collection, pre-amplifier for first stage amplification, the analog-to-digital converter (ADC) for analog speech to digital equivalent conversion, the DSP chip (hardware), the DAC, post amplifier (loud speaker). FIR filter as one of the critical components of hearing aid plays a vital role in the effective operation and performance of the hearing aid device. Therefore the choice of an FIR filter and its design has to be made before incorporating to the algorithm to make it adaptive. There are a thousand and one of FIR filters but Kaiser Window as one of the types was chosen because it uses its empirical method in providing accurate result. Others reasons according to [10] are:

- Kaiser window helps to obtain the filter length of a desired filter in a simple manner. Thus helping to meet the requirement in the specification and thereby achieving the desired frequency response characteristics.
- The shape of the main lobe and side lobes can be determined by manipulating attenuation, \( \alpha \) and filter length, M.
- Kaiser’s approach is referred to as adjustable window. Simply because it provides control over ripple via the addition of another parameter \( \beta \) characterizing the window.

Since the ear is only sensitive to 1 kHz to 4 kHz, a band pass filter of a Kaiser Window FIR type of filter was designed. This implies that only the frequency within this stipulated band will be allowed, others below the pass edge frequency and that above the stop edge frequency will be cut off during sampling. Equation (5) shows the model of the Kaiser Window filter [11].
A. Procedure for Design of Band Pass Filter Using Kaiser Window

The equations required for the design of a bandpass filter is carried out using Kaiser window (\(w_{Kai}(n)\)) as shown in equations 5, 6, 7 to 16.

\[
W_{Kai}(n) = I_0 \left[ \frac{\alpha}{\delta \cdot I_0(\alpha)} \right]_{-M \leq n \leq M} \quad (5)
\]

Where \(W_{Kai}(n)\) = Kaiser Window
\(I_0(\alpha)\) = Bessel’s function
\(I_0(\alpha) = \frac{0.3999 \cdot e^\alpha}{\sqrt{\pi \alpha}} \quad (6)\)

But \(N = 2M\)

Where
\(M = \frac{2.285 \cdot N}{\beta + 0.2233 \cdot N} \quad (7)\)

Where \(N\) is the filter order
\(\alpha\) = attenuation

since \(\beta = \alpha/M\) (5) changes to (8)

\[
W_{Kai}(n) = I_0 \left[ \frac{\alpha}{\delta \cdot I_0(\alpha)} \right]_{-M \leq n \leq M} \quad (8)
\]

Transition Band \(\Delta \omega\) is the band between \(\omega_1\) and \(\omega_2\) given as

\[
\Delta \omega = \omega_2 - \omega_1 \quad (9)
\]

The Normalized cut-off frequency, \(\omega_c\), is the mean of \(\omega_1\) and \(\omega_2\)

\[
\omega_c = \frac{\omega_2 + \omega_1}{2} \quad (10)
\]

\[
\omega_1 = \frac{2\pi f_1}{f_s} \quad (11)
\]

\[
\omega_2 = \frac{2\pi f_2}{f_s} \quad (12)
\]

Where \(f_1\) and \(f_2\) are pass band edge frequency and stop band edge frequency respectively. While \(H_1(dB)\) and \(H_2(dB)\) are gain respectively.

\[
H_1(dB) = \frac{20 \log(1 + \delta_1)}{1 - \delta_1} \quad (13)
\]

\[
H_2(dB) = -20 \log \delta_2 \quad (14)
\]

This implies that the gain is dependent on the pass band and stop band ripple factor.

The ripple factors \((\delta_1, \delta_2)\)

Pass band ripple, \(\delta_1\)

\[
\delta_1 = \frac{10^{\frac{H_1}{10}} - 1}{10^{\frac{H_1}{10}} - 1} \quad (15)
\]

Stop band ripple, \(\delta_2\)

\[
\delta_2 = 10^{\frac{H_2}{20}} - 1
\]

(16) Gives the minimum value of the ripple factor.

<table>
<thead>
<tr>
<th>Table 1: Summary of filter Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass band frequency ((f_1))</td>
</tr>
<tr>
<td>Stop band frequency ((f_2))</td>
</tr>
<tr>
<td>Normalize frequency ((W_1))</td>
</tr>
<tr>
<td>Normalize Frequency ((W_2))</td>
</tr>
<tr>
<td>Normalize Cut-off Frequency ((W_c))</td>
</tr>
<tr>
<td>Pass band ripple</td>
</tr>
<tr>
<td>Stop band ripple</td>
</tr>
<tr>
<td>Sampling Frequency</td>
</tr>
<tr>
<td>Transition Width ((\Delta \omega))</td>
</tr>
<tr>
<td>Gain in pass band ((H_1))</td>
</tr>
<tr>
<td>Gain in stop band ((H_2))</td>
</tr>
<tr>
<td>Filter order, (N)</td>
</tr>
<tr>
<td>Filter Length, (L)</td>
</tr>
<tr>
<td>Adjustable control, (\beta)</td>
</tr>
<tr>
<td>Passband Attenuation, (\delta_1)</td>
</tr>
<tr>
<td>Stop band Attenuation, (\delta_2)</td>
</tr>
</tbody>
</table>

Table 1 shows the specifications of the filter obtained using the equations above. These specifications will then be used in designing the filter using the filter and design analysis (FDA) tool. Setting the values where necessary will give the required result.

Figure 2 shows the filter specification obtained after designing the digital filter in the Filter design and analysis (FDA) tool. Observe that the result obtained is similar to that used in the specification. The displayed information showed that the filter has a direct-form structure, stable and with a filter order of 10. Not only that, the filter also shows that it has a linear phase and as well stable and many other feature associated with FIR filters.
The magnitude response of the filter as indicated in figure 3 shows that only the desired speech within the bandwidth of 1 kHz – 4 kHz will be allowed to pass since the ear is only sensitive to this band. Hence every other side band will be eliminated at shape cut-off. The attenuation at both ends will be attenuated at -15dB. Apart from the magnitude response, every other graphical analysis can be obtained from the menu under analysis such as the filter coefficients, the phase response, pole/zero plots, phase delay among others.

B. Adaptive Algorithm

Adaptive algorithm is a function of the adaptive filter which enables the adaptive filter to adapt both in noisy and noiseless environment. This implies that the filter provides automatic adaptation to the filter parameters of the unknown signal. The flexible nature of adaptive filter is what distinguishes it from conventional analog filters. There are two major adaptive algorithms in use. These are least mean square (LMS) algorithm and Recursive least mean square (RLS) algorithm. RLS algorithm is complex and requires many memories for its implementation. At the other hand, LMS algorithm is regarded as the most populous and commonest algorithm in use because: It is relatively easy to implement in software and hardware due to its computational simplicity and efficient use of memory. Secondly, it performs robustly in the presence of numerical errors caused by finite-precision arithmetic; and finally its behaviour has been analytically characterized to the point where a user can easily set up the system to obtain adequate performance with only limited knowledge about the input and desired response signals.

C. Least Mean Square Algorithm (LMS)

LMS algorithm developed by Windroff and Hoff in 1959 has been defined as an approximation of the steepest descent algorithm, which uses an instantaneous estimate of the gradient vector. This estimate of the gradient vector is based on sample values of the tap input vector and an error signal. Its wide applications have been found in adaptive noise reduction, system identification, adaptive equalization, beam forming, echo cancellation to mention but a few. It is of interest to mention here that the features that attracted the use of the LMS algorithm in the implementation of hearing aid were low computational complexity, proof of convergence in stationary environments and stable behavior when implemented with finite precision arithmetic [12]. The basic idea behind LMS filter is to approach the optimum filter weights by updating the filter weights in a manner to converge to the optimum filter weight [13]. The algorithm is usually initiated by setting the adaptive weights to zero and by finding the mean square error in each step, the next weights are updated. The next update weight equation for LMS algorithm is shown in equations (17) and (18).

\[ W[n + 1] = W[n] - \Delta \mu_e \]  \hspace{1cm} (17)

Where

- \( \mu \) is the step size of the filter.
- If \( \Delta e = e[n] x[n] \), then (17) changes to:

\[ W[n + 1] = W[n] + \mu e[n] x[n] \]  \hspace{1cm} (18)

Equation (18) is the next update weight equation for LMS algorithm.

Where e(n) represents the Mean square error (MSE). The positive sign shows that the MSE gradient would keep increasing positively if the same weight is used for further iteration. But if the sign were negative it would mean that the weight will change to the direction opposite to that of the gradient slope.

D. Summary of LMS Algorithm

The LMS algorithm for Nth order can be summarized below as Parameters:

- Nth = filter order
- \( \mu \) = step size
- Initialization: \( x(0) = w(0) = [0 \ 0 \ldots 0]^T \)
- Computation: For \( n = 0, 1, 2, \ldots \)
  - \( X(n) = [x(n), x(n - 1), \ldots, x(n - N + 1)]^T \)
  - \( e(n) = d(n) - W[n]^T x(n) \)
  - \( W(n+1) = W(n) + \mu e(n) x(n) \)

E. Normalized Least Mean Square Algorithm (NLMS)

The NLMS algorithm is an extension of LMS algorithm, a variant of LMS algorithm. According to research, LMS algorithm has been found to be more stable, robust and adaptable. The main drawback of the “pure” LMS algorithm is that it is sensitive to the scaling of its input [14]. This makes it very hard to choose a learning rate \( \mu \) that guarantees stability of the algorithm. Other preferences of NLMS algorithm over LMS algorithm include:

- The LMS algorithm experiences gradient noise amplification when the convergence factor \( \mu \) is large. This effect can be resolved in NLMS algorithm by normalizing the step size.
As a result of the normalization of the step size, NLMS algorithm experiences a faster convergence of the output signal than LMS algorithm since it utilizes a variable convergence factor aiming at minimization of the instantaneous output error.

The adaptation constant for NLMS is dimensionless while that of the LMS has the dimension of inverse power. Consequently, the derivation of NLMS algorithm is similar to that of LMS the only difference is the normalization of the step size as demonstrated below.

\[ W[n+1] = W[n] + \mu e[n]x[n] \] % update weight adaptation for LMS algorithm

The normalization of the LMS algorithm gives rise to the update equation of NLMS algorithm.

\[ W[n+1] = W[n] + \frac{\mu e[n]x[n]}{\gamma + x^H[n]x[n]} \] % Update Weight adaptation for NLMS algorithm

\( \gamma \) is a constant term for normalization and is always less than 1. The inclusion of the parameter, \( \gamma \), in the coefficient update is to avoid large step size when \( x^H(n) x(n) \) becomes small or to avoid division by zero in normalization operation.

**F. Summary of NLMS Algorithm**

Parameters: \( N \) = filter order
\( \mu \) = step size

Initialization:
\( x(0) = w(0) = [0 0 \ldots 0]^H \)
Choose \( \mu_n \) in the range \( 0 < \mu_n \leq 2 \)
\( \gamma \) = small constant

Computation for \( n \geq 0 \)
\( x(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^H \)
\( e(n) = d(n) - x(n)w^H(n) \)

\[ W[n+1] = W[n] + \frac{\mu e[n]x[n]}{\gamma + x^H[n]x[n]} \]

**IV. SIMULATION RESULT, ANALYSIS AND DISCUSSIONS**

In this section, the filter output of the previous model of LMS algorithm in figure 4 was used to compare and evaluate the filter output of the newly developed model of NLMS algorithm as shown in figure 3. Both algorithms were both carried out in Simulink. The design procedure followed the specifications show cased in figure 1. This was made possible by the signal processing block sets provided from Simulink library. The design was initiated by generating an input signal, \( x(n) \) using the sine wave signal block. The parameter of the block was set at 100Hz frequency, 0.001ms of sampling time and amplitude at 1. The noise signal, \( s(n) \) was thereafter generated using the random source block, setting the source type to Gaussian distribution.

When this was completed, the band pass FIR filter designed using filter design and analysis tool (FDA tool) was later incorporated. And thereafter, the NLMS block was added and used respectively to generate NLMS and LMS algorithm and shown in figure 3 & 4. The essence of the time scope block is to enable the display of the filter output signal, the corrupted signal and the error signal graphs. Each of these graphs shows the behaviour of the filter when the step size was varied between 0 and 2 while keeping the filter length constant. Finally, the display block enabled the value of the adaptive weight at any given time to be accounted for. The simulation results therefore illustrates the adaptation of the weight of adaptive filter used in hearing aid application as the user changes from one environment to another. The output and error signal in fig 3 and fig 4 shall be the area of concentration in evaluating for improvement in mean square noise reduction and faster speed of convergence.

**Figure 3: Simulink block for NLMS based FIR Filter**

**Figure 4: Simulink Block Model for LMS algorithm**

The simulation parameters used in both cases include Filter order, \( N = 10 \), since filter length, \( L = N+1 \). Then \( L = 11 \), leakage factor = 1.0, sampling time, \( t_s = 0.001s \), Initial filter weight = 0, step size, \( \mu = [0.005, 0.05, 0.08, 1.0] \).
Simulation Result for NLMS

Fig 5(a): NLMS filter output when step size = 0.005

Fig 5(b): NLMS filter output when step size = 0.05

Fig 5(c): NLMS filter output when step size = 0.08

Fig 5(d): NLMS filter output when step size = 1.0

Simulation Result for LMS

Fig 5(e): LMS filter output when step size = 0.005

Fig 5(f): LMS filter output when step size = 0.05

Fig 5(g): LMS filter output when step size = 0.08

Fig 5(h): LMS filter output when step size = 1.0
Table 2: Summary of Simulation Result

<table>
<thead>
<tr>
<th>Step Size</th>
<th>NLMS(ms)</th>
<th>LMS(ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>7.0-8.0</td>
<td>8.00-9.50</td>
</tr>
<tr>
<td>0.05</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>0.08</td>
<td>1.5-2.0</td>
<td>5.5</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
<td>No Simulation</td>
</tr>
</tbody>
</table>

Table 2 shows that the time taken for each of the algorithms to converge as the step size was varied from 0 – 1.0. NLMS from the table showed a reduction in the time taken for the algorithm to converge as the step size is varied progressively. Also, in LMS algorithm the time reached a certain time of 5.5ms and maintained its time until it met its performance limit. Consequently, NLMS simulation provided graphs from figure 5a to 5g. Each of these graphs contains an input signal, the corrupted signal, the filtered output signal and the error signal. Observation from the results proved that NLMS algorithm showed superiority over LMS algorithm as the step size value is varied beyond 1.0 as shown in figure 5d and 5g.

The time taken by NLMS algorithm to converge decreases as the step size increases, even though it maintained the same time, 5.5ms with LMS algorithm when the step size was at 0.05. Above 0.08 step size value, the algorithm converged before zero milliseconds. Also notice that LMS algorithm maintained a constant time with no further reduction in the time of convergence as the step size increases. When the step size was increased to 1.0 NLMS algorithm was observed to have converged before approaching 0sec, while LMS algorithm could not simulate further or show any output. At this juncture a distinctive line was drawn between both algorithms. This simply indicates the limitation of the performance of LMS algorithm while NLMS algorithm was able to overcome the limitation of the LMS algorithm by converging faster as it approaches the step size of 1.0. Indeed, this is an improvement. At this point, it continued to show a faster speed of convergence while maintaining a minimum mean square reduction as the step size increased within the bound of 0 < μ < 2. Notice in fig 5(d) that the input signal was approximately equal to output (filtered) signal showing that the error has been drastically minimized.

Table 3 displays the value of the adaptive weight of the filter at any given value of the step size. These weights are mere update of the filter coefficient as the filter experiences changes in its background. During the simulation, it was observed that these adaptive weights were not constant. That is the value keep changing at any time the simulation was run. This simply describes the dynamics of the filter coefficient which depends on the environment where the simulation was undertaken. The implication of the result is that the faster the convergence rate of the algorithm the shorter the time taken to offset noise in the aid. This means that the user of the aid will not notice what happened since the time taken to carry out the operation is in milliseconds. Also notice that since LMS algorithm could not simulate at step size of 1.0, no filter output produced and no corresponding value for the adaptive weight. Hence cannot give better result/ performance.

V. CONCLUSION

The performance evaluation of both algorithms for improvement of the digital hearing aid using a Matlab simulator was conducted successfully. During the simulation, NLMS algorithm showed a reduction in the time. This explains the high speed of algorithm convergence. During this period, the desired signal is approximately equal to the output signal. The results of the simulation clearly indicated that the performance of the new developed (NLMS) algorithm performed better than the previous (LMS) algorithm in terms of speed of convergence and ability to maintain minimum mean square error.

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